Journal of Applied Mechanics and Technical Physics, Vol. 49, No. 6, pp. 1047–1054, 2008

WAVE PROCESSES IN SOLIDS WITH DEFECTS

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UDC 539

The propagation of plane harmonic waves in viscoelastic and elastoviscoplastic materials are studied using the equations of the field theory of defects, the kinematic identities for an elastic continuum with defects, and the dynamic equations of gauge theory. Wave propagation velocities and refraction and absorption coefficients are determined. The structure of the waves and the correlation between the displacement waves and the defect-field waves determining plastic deformation are analyzed.

Key words: *defect field theory, elastoviscoplastic materials, wave structure.*

Introduction. The inelastic behavior of deformable solids has long been an important problem in mechanics and physics. This is because the region of elastic deformations is bounded and many processes that are important for the operation and production of new materials and products occur beyond this region. Among the practically significant phenomena observed outside the region of elastic deformation are plasticity, fracture, creep, hardening, etc. In the present paper, the dynamic features of inelastic deformation due to translation defects are considered using the continual description traditional for continuum mechanics. As is known, the dynamics of translation defects determines dislocation plasticity, which is one of the mechanisms of inelastic behavior of materials, along with mechanical twinning, martensite inelasticity, and disclination plasticity [1]. The wave solutions of the equations of the field theory of defects are used. The equations of the field theory of defects include the kinematic identities of the continual theories of defects [2] and the dynamic equations of the gauge theory [3]. In the present paper, we consider viscoelastic, elastoviscoplastic, and viscoplastic materials, compare the corresponding wave solutions, and analyze the structure of the waves considered. Arbitrary dynamic solutions of linear systems can be represented in the form of a superposition of waves.

1. Mathematical Model. The equations of the field theory of defects

$$B\,\frac{\partial}{\partial x_i}\,I_{ij} = -P_j, \qquad e_{ikl}\,\frac{\partial}{\partial x_k}\,I_{lj} = \frac{\partial}{\partial t}\,\alpha_{ij},$$
$$\frac{\partial}{\partial x_k}\,\alpha_{ki} = 0, \qquad S e_{ikl}\,\frac{\partial}{\partial x_k}\,\alpha_{lj} = -B\,\frac{\partial}{\partial t}\,I_{ij} - \sigma_{ij} \tag{1}$$

obtained using the gauge approach [3] are the basic equations in the analysis of the dynamics of defect fields in various materials. Here α_{ij} and I_{ij} are the dislocation density tensor and the dislocation flux density tensor, σ_{ij} and P_i are the effective stress and momentum, and B and S are constants of the theory. In the continual theory of defects [2, 4], the characteristics of the dislocation ensemble I_{ij} and α_{ij} are defined by the plastic distorsion gradients β_{ij}:

$$I_{ij} = -\frac{\partial \beta_{ij}}{\partial t}, \qquad \alpha_{ij} = -e_{ikn}\,\frac{\partial \beta_{nj}}{\partial x_k}.$$

The physical meaning of these quantities is clear from the following expressions:

$$\alpha_{ij} = \frac{db_j}{ds_i}, \qquad I_{ij} = \frac{d}{dl_i}\,\frac{\partial b_j}{\partial t}.$$

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0021-8944/08/4906-1047

Here $\boldsymbol{b}$ is the total Burgers vector of all dislocations crossing an oriented unit area s bounded by a contour with the unit tangential vector $\boldsymbol{l}$. The quantities σ_{ij} and P_i satisfy the dynamic equilibrium equation

$$\frac{\partial P_i}{\partial t} = \frac{\partial \sigma_{ki}}{\partial x_k}, \tag{2}$$

which is the condition of compatibility of system (1). Equations (1) and (2) were used to study the propagation of plane waves of defect fields in homogeneous viscoplastic materials [5, 6] (and in the presence of interfaces [7, 8]) given by the relation

$$\sigma_{ij} = \theta_{ijkl} I_{kl}$$

(θ_{ijkl} is the viscosity-coefficient tensor). Dispersion relations are obtained for viscoelastic materials [9] described by the equation

$$\sigma_{ij} = \sigma_{ij}^{\rm el} + \sigma_{ij}^{v}. \tag{3}$$

Here $\sigma_{ij}^{\rm el}$ and σ_{ij}^{v} are the elastic and viscous stresses expressed in terms of the components of the displacement vector U_i, the elastic modulus tensor C_{ijkl}, and the viscosity coefficient tensor η_{ijkl} [4]:

$$\sigma_{ij}^{\rm el} = C_{ijkl}\,\partial_k U_l, \qquad \sigma_{ij}^{v} = \eta_{ijkl}\,\partial_k V_l, \qquad V_i = \frac{\partial U_i}{\partial t}. \tag{4}$$

The momentum of the viscoelastic material is defined in the standard manner:

$$P_i = \rho V_i \tag{5}$$

(ρ is the density of the material). We perform a similar investigation for elastoviscoplastic materials [10] described by the equation

$$\sigma_{ij} = \sigma_{ij}^{\rm el} + \sigma_{ij}^{\theta}, \tag{6}$$

where

$$\sigma_{ij}^{\rm el} = C_{ijkl}\,\partial_k U_l, \qquad \sigma_{ij}^{\theta} = \theta_{ijkl} I_{kl}, \qquad P_i = \rho(\partial U_i/\partial t).$$

In the case of a homogeneous isotropic solid, the tensors of the elastic moduli and viscosity coefficients (4) and (6) are written as

$$\begin{aligned} C_{ijkl} &= \lambda\delta_{ij}\delta_{kl} + \mu(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}), \\ \eta_{ijkl} &= \xi\delta_{ij}\delta_{kl} + \gamma(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}); \end{aligned} \tag{7}$$

$$\theta_{ijkl} = \pi\delta_{ij}\delta_{kl} + \zeta(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}), \tag{8}$$

where λ and μ are the Lamé coefficients, ξ and γ are the bulk and shear viscosities of the elastic solid, π and ζ are the bulk and shear viscosities of the plastically deformed solid, and δ_{ij} are the Kronecker symbols. The definitions of various materials used in this work are discussed in detail in [10, 11]. Materials that exhibit viscosity properties in the elastic region are referred to as viscoelastic materials. Materials that exhibit viscosity properties only in the plastic deformation region are referred to as elastoviscoplastic materials. Such materials are elastic until the attainment of the plastic state. Elastoviscoplastic materials exhibiting viscous properties in the elastic and plastic regions have the most general properties [10, 11]. The energy dissipation mechanisms in elastic and plastic deformations are different [12]. As a result, the viscosity for elastic and plastic deformation differ by 5–7 orders of magnitude. The plastic flow viscosity, which reaches values of 10^{13}–10^{15} Pa · sec, is much higher than the viscosity calculated from the elastic vibration law and is equal to 10^7–10^8 Pa · sec.

2. Dispersion Relations. We write solutions of system (1) in the form

$$\{\alpha_{ij}(r,t), I_{ij}(r,t), U_i(r,t)\} = [\alpha_{ij}(x), I_{ij}(x), U_i(x)] \exp(-i\omega t),$$

assuming that the unknown quantities depend on one coordinate x. For the complex components $\alpha_{ij}(x)$, $I_{ij}(x)$, and $U_i(x)$, Eqs. (1) are written as

$$B\,\partial_x I_{xj}(x) = i\omega\rho U_j(x); \tag{9}$$

$$i\omega\alpha_{xj}(x) = 0, \qquad i\omega\alpha_{yj}(x) = \partial_x I_{zj}(x), \qquad i\omega\alpha_{zj}(x) = -\partial_x I_{yj}(x); \tag{10}$$

$$\partial_x\alpha_{xj}(x) = 0; \tag{11}$$

$$\begin{gathered} i\omega B I_{xj}(x) - \sigma_{xj}(x) = 0, \qquad i\omega B I_{yj}(x) - \sigma_{yj}(x) = -S\,\partial_x\alpha_{zj}(x), \\ i\omega B I_{zj}(x) - \sigma_{zj}(x) = S\,\partial_x\alpha_{yj}(x). \end{gathered} \tag{12}$$

In the case of elastoviscoplastic solids, we assume for simplicity that the viscosity tensor (8) is a constant θ. Under this assumption, the stress tensor components (6) become

$$\begin{array}{ccc} (\lambda+2\mu)\,\partial_x U_x(x) + \theta I_{xx}(x), & \mu\,\partial_x U_y(x) + \theta I_{xy}(x), & \mu\,\partial_x U_z(x) + \theta I_{xz}(x), \\ \mu\,\partial_x U_y(x) + \theta I_{yx}(x), & \lambda\,\partial_x U_x(x) + \theta I_{yy}(x), & \theta I_{yz}(x), \\ \mu\,\partial_x U_z(x) + \theta I_{zx}(x), & \theta I_{zy}(x), & \lambda\,\partial_x U_x(x) + \theta I_{zz}(x). \end{array} \tag{13}$$

Using the first relation in (12), the expressions for the stress tensor components (13), and equality (9), we obtain the following equations for the dynamics of the displacement vector components $U_i(x)$:

$$\begin{gathered} \partial_x^2 U_x(x) + (\omega/C_1)^2(1 + i\theta/(B\omega))U_x(x) = 0, \\ \partial_x^2 U_y(x) + (\omega/C_2)^2/(1 + i\theta/(B\omega))U_y(x) = 0, \\ \partial_x^2 U_z(x) + (\omega/C_2)^2/(1 + i\theta/(B\omega))U_z(x) = 0, \end{gathered} \tag{14}$$

where $C_1 = \sqrt{(\lambda+2\mu)/\rho}$ and $C_2 = \sqrt{\mu/\rho}$ are the longitudinal and transverse velocities of elastic waves. The solutions of (14) are known:

$$\begin{gathered} U_x(x) = a_1\exp(ik_1x) + b_1\exp(-ik_1x), \\ U_y(x) = a_2\exp(ik_2x) + b_2\exp(-ik_2x), \\ U_z(x) = a_3\exp(ik_2x) + b_3\exp(-ik_2x). \end{gathered}$$

Here

$$k_1^2 = (\omega/C_1)^2(1 + i\theta/(B\omega)), \qquad k_2^2 = (\omega/C_2)^2(1 + i\theta/(B\omega)), \tag{15}$$

and a_1, a_2, a_3, b_1, b_2, and b_3 are unknown constants determined from the boundary conditions. Below, for brevity, we use expressions of the form $a_1(b_1)\exp(\pm ik_1x)$ to denote the sum $a_1\exp(ik_1x) + b_1\exp(-ik_1x)$. Expressions (15) can be written as

$$k_1 = \omega(n + i\chi)/C_1, \qquad k_2 = \omega(n + i\chi)/C_2, \tag{16}$$

if we introduce the refraction and absorption coefficients n and χ [13], which are related to the quantity $\tan\delta = \theta/(B\omega)$, called the loss factor:

$$n = [((\tan^2\delta + 1)^{1/2} + 1)/2]^{1/2}, \qquad \chi = [((\tan^2\delta + 1)^{1/2} - 1)/2]^{1/2}.$$

The refraction coefficient n determines the wavephase velocity, and the absorption coefficient χ characterizes the rate of decrease in the wave amplitude in the wave propagation direction.

Knowing $U_i(x)$, from the first equality in (12), it is possible to find the components of the dislocation flux density tensor $I_{xj}(x)$:

$$\begin{gathered} I_{xx}(x) = (\rho\omega/(Bk_1))[a_1(-b_1)\exp(\pm ik_1x)], \\ I_{xy}(x) = (\rho\omega/(Bk_2))[a_2(-b_2)\exp(\pm ik_2x)], \\ I_{xz}(x) = (\rho\omega/(Bk_2))[a_3(-b_3)\exp(\pm ik_2x)]. \end{gathered} \tag{17}$$

We obtain equations for $I_{yj}(x)$ and $I_{zj}(x)$ using the second and third equalities in (10) and (12):

$$\begin{gathered} -\omega^2 B I_{yj}(x) - i\omega\sigma_{yj}(x) = S\,\partial_x^2 I_{yj}(x), \\ -\omega^2 B I_{zj}(x) - i\omega\sigma_{zj}(x) = S\,\partial_x^2 I_{zj}(x). \end{gathered} \tag{18}$$

Taking into account (13), we write the first equality in (18) as follows:

$$\partial_x^2 I_{yx}(x) + k_3^2 I_{yx}(x) = -(i\omega\mu/S)\,\partial_x U_y(x),$$
$$\partial_x^2 I_{yy}(x) + k_3^2 I_{yy}(x) = -(i\omega\lambda/S)\,\partial_x U_x(x), \tag{19}$$
$$\partial_x^2 I_{yz}(x) + k_3^2 I_{yz}(x) = 0.$$

Here

$$k_3^2 = (\omega/C_3)^2(1 + i\theta/(B\omega)), \qquad k_3 = \omega(n + i\chi)/C_3, \qquad C_3 = \sqrt{S/B}. \tag{20}$$

From the second equality in (18), it follows that the equations for I_{zz} and I_{zy} coincide with the equations for I_{yy} and I_{yz} in (19), and the component I_{zx} can be determined from the equation for I_{yx} by replacing U_y by U_z. The solutions of the equations for I_{yz} and I_{zy} are written as

$$I_{yz}(x) = q_3(d_3)\exp(\pm ik_3x), \qquad I_{zy}(x) = q_5(d_5)\exp(\pm ik_3x), \tag{21}$$

where q_3, q_5, d_3, and d_5 are constants. The expressions for the remaining components of the dislocation flux density contain two terms, one of which is a solution of the homogeneous equation (19) and the other is determined by the form of the function on the right side of the equation

$$I_{yx}(x) = q_1(d_1)\exp(\pm ik_3x) + [\omega\mu k_2/(S(k_3^2 - k_2^2))]a_2(-b_2)\exp(\pm ik_2x),$$
$$I_{yy}(x) = q_2(d_2)\exp(\pm ik_3x) + [\omega\lambda k_1/(S(k_3^2 - k_1^2))]a_1(-b_1)\exp(\pm ik_1x),$$
$$I_{zx}(x) = q_4(d_4)\exp(\pm ik_3x) + [\omega\mu k_2/(S(k_3^2 - k_2^2))]a_3(-b_3)\exp(\pm ik_2x), \tag{22}$$
$$I_{zz}(x) = q_6(d_6)\exp(\pm ik_3x) + [\omega\lambda k_1/(S(k_3^2 - k_1^2))]a_1(-b_1)\exp(\pm ik_1x)$$

(q_1, q_2, q_4, q_6, d_1, d_2, d_4, and d_6 are unknown constants). As regards the dislocation density tensor components $\alpha_{xj}(x)$, the first equalities in (10) and (11) imply that

$$\alpha_{xj}(x) \equiv 0.$$

The components $\alpha_{yj}(x)$ and $\alpha_{zj}(x)$ given in Table 1 can be obtained from the second and third equalities in (10).

In the case of a homogeneous isotropic viscoelastic solid defined by formulas (3) and (4) with (7) taken into account, the tensor stress components are written as follows:

$$\begin{array}{ccc} [(\lambda+2\mu)+(\xi+2\gamma)\partial_t]\,\partial_x U_x(x), & (\mu+\gamma\,\partial_t)\,\partial_x U_y(x), & (\mu+\gamma\,\partial_t)\,\partial_x U_z(x), \\ (\mu+\gamma\,\partial_t)\,\partial_x U_y(x), & (\lambda+\xi\,\partial_t)\,\partial_x U_x(x), & 0, \\ (\mu+\gamma\,\partial_t)\,\partial_x U_z(x), & 0, & (\lambda+\xi\,\partial_t)\,\partial_x U_x(x). \end{array} \tag{23}$$

The components of the displacement vector $U_i(x)$ and the components of the dislocation flux density tensor $I_{xj}(x)$ obtained on the basis of the first relation in (12), (9), and (23) become

$$U_x(x) = A_1(B_1)\exp(\pm iK_1x),$$
$$U_y(x) = A_2(B_2)\exp(\pm iK_2x),$$
$$U_z(x) = A_3(B_3)\exp(\pm iK_2x),$$
$$I_{xx} = (\rho\omega/(BK_1))A_1(-B_1)\exp(\pm iK_1x),$$
$$I_{xy} = (\rho\omega/(BK_2))A_2(-B_2)\exp(\pm iK_2x),$$
$$I_{xz} = (\rho\omega/(BK_2))A_3(-B_3)\exp(\pm iK_2x).$$

Here

$$K_1^2 = (\omega/C_1)^2/(1 - i\omega(\xi+2\gamma)/(\lambda+2\mu)), \qquad K_2^2 = (\omega/C_2)^2/(1 - i\omega\gamma/\mu), \tag{24}$$

TABLE 1

$\alpha_{ij}(x)$	Elastoviscoplastic material $[\alpha_{ij}(x) = l\exp(\pm ik_3x) + f(x)]$		Viscoelastic material $[\alpha_{ij}(x) = L\exp(\pm iK_3x) + F(x)]$	
	l	$f(x)$	L	$F(x)$
$\alpha_{yx}(x)$	$\dfrac{q_4(-d_4)}{V_3}$	$\dfrac{\mu a_3(b_3)\exp(\pm ik_2x)}{S(C_3^2/C_2^2-1)}$	$\dfrac{Q_4(-D_4)}{C_3}$	$-\dfrac{\omega^2\rho A_3(B_3)\exp(\pm iK_2x)}{S(K_3^2-K_2^2)}$
$\alpha_{yy}(x)$	$\dfrac{q_5(-d_5)}{V_3}$	—	$\dfrac{Q_5(-D_5)}{C_3}$	—
$\alpha_{yz}(x)$	$\dfrac{q_6(-d_6)}{V_3}$	$\dfrac{\lambda a_1(b_1)\exp(\pm ik_1x)}{S(C_3^2/C_1^2-1)}$	$\dfrac{Q_6(-D_6)}{C_3}$	$\dfrac{\omega^2\rho A_1(B_1)\exp(\pm iK_1x)}{SK_4^2(K_3^2/K_1^2-1)}$
$\alpha_{zx}(x)$	$\dfrac{(-q_1)d_1}{V_3}$	$-\dfrac{\mu a_2(b_2)\exp(\pm ik_2x)}{S(C_3^2/C_2^2-1)}$	$\dfrac{(-Q_1)D_1}{C_3}$	$-\dfrac{\omega^2\rho A_2(B_2)\exp(\pm iK_2x)}{S(K_3^2-K_2^2)}$
$\alpha_{zy}(x)$	$\dfrac{(-q_2)d_2}{V_3}$	$-\dfrac{\lambda a_1(b_1)\exp(\pm ik_1x)}{S(C_3^2/C_1^2-1)}$	$\dfrac{(-Q_2)D_2}{C_3}$	$-\dfrac{\omega^2\rho A_1(B_1)\exp(\pm iK_1x)}{SK_4^2(K_3^2/K_1^2-1)}$
$\alpha_{zz}(x)$	$\dfrac{(-q_3)d_3}{V_3}$	—	$\dfrac{(-Q_3)D_3}{C_3}$	—

C_1 and C_2 are the longitudinal and transverse velocities of elastic waves and A_1, A_2, A_3, B_1, B_2, and B_3 are unknown constants determined from the boundary conditions. Introducing the refraction coefficients n_1 and n_2 and the absorption coefficients χ_1 and χ_2, related to the quantities $\tan\delta_1 = \omega(\xi + 2\gamma)/(\lambda + 2\mu)$ and $\tan\delta_2 = \omega\gamma/\mu$, which are called loss factors, we can write expressions (24) as

$$K_1 = \omega(n_1 + i\chi_1)/C_1, \qquad K_2 = \omega(n_2 + i\chi_2)/C_2. \tag{25}$$

The refraction and absorption coefficients are expressed in terms of the loss factors:

$$n_1 = \Big(\frac{(\tan^2\delta_1 + 1)^{1/2} + 1}{2(\tan^2\delta_1 + 1)}\Big)^{1/2}, \qquad \chi_1 = \Big(\frac{(\tan^2\delta_1 + 1)^{1/2} - 1}{2(\tan^2\delta_1 + 1)}\Big)^{1/2}. \tag{26}$$

The relations for n_2 and χ_2 are similar. In the case of a viscoelastic solid, the first equation in (18) can be written as

$$\begin{gathered} \partial_x^2 I_{yx}(x) + K_3^2 I_{yx}(x) = -(i\omega/S)(\mu - i\omega\gamma)\,\partial_x U_y(x) = -(i\omega^3\rho/(SK_2^2))\,\partial_x U_y(x), \\ \partial_x^2 I_{yy}(x) + K_3^2 I_{yy}(x) = -(i\omega/S)(\lambda - i\omega\xi)\,\partial_x U_x(x) = -(i\omega^3\rho/(SK_4^2))\,\partial_x U_x(x), \\ \partial_x^2 I_{yz}(x) + K_3^2 I_{yz}(x) = 0. \end{gathered} \tag{27}$$

From the second equality in (18), we obtain equations for the components I_{zz} and I_{zy} which coincide with Eqs. (27) for I_{yy} and I_{yz}, and the component I_{zx} can be found from the equation for I_{yx} by replacing U_y by U_z. In Eqs. (27), the following notation is used:

$$K_3^2 = (\omega/C_3)^2, \quad C_3 = \sqrt{S/B}, \quad K_4^2 = (\omega/C_4)^2/(1 - i\omega\xi/\lambda), \quad C_4 = \sqrt{\lambda/\rho}.$$

In the case of a viscoelastic solid, solutions (27) for the components of the dislocation flux density tensor are similar to (21) and (22) since

$$\begin{gathered} I_{yx}(x) = Q_1(D_1)\exp(\pm iK_3x) + [\omega^3\rho/(SK_2(K_3^2 - K_2^2))]A_2(-B_2)\exp(\pm iK_2x), \\ I_{yy}(x) = Q_2(D_2)\exp(\pm iK_3x) + [\omega^3\rho K_1/(SK_4^2(K_3^2 - K_1^2))]A_1(-B_1)\exp(\pm iK_1x), \\ I_{yz}(x) = Q_3(D_3)\exp(\pm iK_3x), \\ I_{zx}(x) = Q_4(D_4)\exp(\pm iK_3x) + [\omega^3\rho/(SK_2(K_3^2 - K_2^2))]A_3(-B_3)\exp(\pm iK_2x), \end{gathered} \tag{28}$$

$$I_{zy}(x) = Q_5(D_5)\exp(\pm iK_3x),$$

$$I_{zz}(x) = Q_6(D_6)\exp(\pm iK_3x) + [\omega^3\rho K_1/(SK_4^2(K_3^2 - K_1^2))]A_1(-B_1)\exp(\pm iK_1x).$$

Here Q_1, Q_2, Q_4, Q_6, D_1, D_2, D_4, and D_6 are unknown constants. The dislocation density tensor components given in Table 1 can be determined on the basis of (10), (11), and (28).

3. Wave Structure. We study the structure of the waves propagating in various materials with defects. From the first equality in (10) and (11), it follows that the projection of the dislocation flux density tensor onto the wave propagation direction is identically equal to zero. The dislocation density wave is transverse, and the oscillations of the nonzero components α_{ij} occur in its plane. The projections of the defect density waves onto the wave propagation direction in elastoviscoplastic (13) and viscoelastic (23) materials are determined by the dynamics of the gradients of the displacement vector components:

$$I_{xx}(x) = -i(\rho\omega/(Bk_1^2))\,\partial_x U_x(x), \qquad I_{xy}(x) = -i(\rho\omega/(Bk_2^2))\,\partial_x U_y(x),$$

etc. Here k_n is the wave vector of the corresponding displacement component (15), which in the case of viscoelastic materials should be replaced by K_n (24). The components I_{yz} and I_{zy} do not depend on displacement waves since

$$I_{yz} \sim \exp[-i\omega t(t \pm x/V_3)], \qquad I_{zy} \sim \exp[-i\omega t(t \pm x/V_3)], \tag{29}$$

where $V_3 = C_3$ for viscoelastic materials and $V_3 = C_3/\sqrt{1 + i\theta/(B\omega)}$ for elastoviscoplastic materials. The waves of the remaining components of the dislocation flux density tensor contain the contribution due to the fluctuations of the displacement vector. The diagonal components I_{yy} and I_{zz} depend and the natural oscillations of the dislocation ensemble and on the longitudinal displacement gradient:

$$I_{yy} \sim \exp[-i\omega t(t \pm x/V_3)] + \partial_x U_x, \qquad I_{zz} \sim \exp[-i\omega t(t \pm x/V_3)] + \partial_x U_x. \tag{30}$$

The transverse displacement gradients influence the propagation of the components I_{yx} and I_{zx}:

$$I_{yx} \sim \exp[-i\omega t(t \pm x/V_3)] + \partial_x U_y, \qquad I_{zx} \sim \exp[-i\omega t(t \pm x/V_3)] + \partial_x U_z. \tag{31}$$

Using (29)–(31), from (10) and (11) we can express the transverse components of the dislocation density tensor as

$$\alpha_{yy} \sim \exp[-i\omega t(t \pm x/V_3)]/V_3, \qquad -\alpha_{zz} \sim \exp[-i\omega t(t \pm x/V_3)]/V_3,$$

$$\alpha_{yx} \sim \exp[-i\omega t(t \pm x/V_3)]/V_3 + \partial_x^2 U_z/(i\omega),$$

$$\alpha_{zx} \sim -\exp[-i\omega t(t \pm x/V_3)]/V_3 - \partial_x^2 U_y/(i\omega),$$

$$\alpha_{yz} \sim \exp[-i\omega t(t \pm x/V_3)]/V_3 + \partial_x^2 U_x/(i\omega),$$

$$-\alpha_{zy} \sim \exp[-i\omega t(t \pm x/V_3)]/V_3 + \partial_x^2 U_x/(i\omega).$$

Conclusions. From the obtained solutions of the equations of the field theory of defects, it follows that the plane harmonic waves propagating in materials with defects are of different nature: elastic-displacement waves, defect-density waves, and defect flux density waves. The defect density waves determine the dynamics of long-range internal stress, and the defect flux density waves plastic distortion [14]. Wave propagation in different materials has a number of characteristic features. In the examined model of elastoviscoplastic materials, all waves (of the elastic continuum and defect continuum) are characterized by the same refraction and absorption coefficient but differ in the oscillation propagation velocity (16), (20). In viscoelastic materials, all wave propagation velocities are also different. The elastic wave velocities (26) which have complex values are determined by sound velocities and refraction and absorption coefficients dependent on the diagonal or shear components of the viscosity coefficient tensor (25). The propagation velocity of dislocation ensemble oscillations in viscoelastic materials is real and is expressed in terms of the defect field constants (28).

For both materials considered, it is shown that the defect density waves are transverse. This implies that the defect density tensor components whose first index corresponds to the wave propagation direction are equal to zero. This property, established on the basis of equalities (10) and (11) the kinematic identities of the continual theories of defects, is valid for any materials with defects considered within the framework of the continual approach. From a viewpoint of material microstructure, the transverse character of the defect density waves implies that, for all dislocations crossing an infinitesimal area perpendicular to the wave propagation direction, the total Burgers vector is equal to zero. As regards defect flux density waves, the character of their propagation depends on the properties of the material specified by material relations. In viscoelastic and elastoviscoplastic materials, the waves of the longitudinal components of the dislocation flux density tensor and the displacement waves are interrelated, unlike in viscoplastic materials [5–8], where the defect flux density waves are transverse. As in the case of viscoplastic materials [5–7], whose waves are characterized by the unique wave vector (20), for weakly damped waves propagating in elastoviscoplastic materials at $\tan\delta \ll 1$, when

$$n = 1 = \text{const}, \qquad \chi = (1/2)\tan\delta = \theta/(2B\omega),$$

dispersion is absent and the dissipation is frequency dependent. For weakly damped waves in viscoelastic materials at $\tan\delta_1 \ll 1$ and $\tan\delta_2 \ll 1$, when

$$n_1 = 1, \qquad \chi_1 = (1/2)\tan\delta_1 = \omega(\xi + 2\gamma)/(2(\lambda + 2\mu)),$$

$$n_2 = 1, \qquad \chi_2 = (1/2)\tan\delta_2 = \omega\gamma/(2\mu),$$

dispersion is also absent, and the dissipation is frequency is dependent. For strongly damped waves in elastoviscoplastic materials at $\tan\delta \gg 1$ and in viscoelastic materials at $\tan\delta_1 \gg 1$ and $\tan\delta_2 \gg 1$, frequency-dependent dispersion and dissipation take place because in these materials

$$n \approx \chi = \sqrt{(1/2)\tan\delta} = \sqrt{\theta/(2B\omega)},$$

$$n_1 \approx \chi_1 = 1/\sqrt{2\tan\delta_1} = \sqrt{(\lambda + 2\mu)/2\omega(\xi + 2\gamma)},$$

$$n_2 \approx \chi_2 = 1/\sqrt{2\tan\delta_2} = \sqrt{\mu/(2\omega\gamma)}.$$

In the case of strongly damped wave, the process does not occur since the waves are damped at very small distances compared to the wavelength λ:

$$d = C/(\omega\chi) = \lambda/(2\pi\chi).$$

The results allow one to analyze the dynamics of waves in elastoviscoplastic materials having viscous properties in elastic and plastic deformations and can be used in indestructible control methods, seismic engineering, etc., where it is necessary to take into account the dynamic effects of inelastic deformation determined by the defect structure of the material.

This work was supported by the Russian Foundation for Basic Research (Grant No. 05-01-00303).

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